THE
MATHEMATICAL GAZETTE.
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THE CORRELATION OF MATHEMATICAL AND SCIENCE TEACHING.*

REPORT OF A JOINT COMMITTEE OF THE MATHEMATICAL ASSOCIATION AND THE ASSOCIATION OF PUBLIC SCHOOL SCIENCE MASTERS.

Several societies have during the past few years considered the teaching of mathematics and of science; since, however, they have confined their attention to the subjects separately, and in many instances to particular types of school, the Joint Committee decided not to rely upon data previously collected, but to send a circular to all the schools mentioned in the *Schoolmasters' Year Book*, asking for details of the conditions under which the subjects are at present taught. This has been done and replies received from about 300 schools representing all types of non-primary education: a summary of the replies will be found in Appendix B, and the Committee wish to express their thanks to those gentlemen who, by so fully answering the questions asked, have made it possible to present a report dealing with a larger number of schools than has been hitherto possible.

The Committee wish to make several recommendations; they however recognise with pleasure that the returns prove not only that a considerable advance has been made during the past ten years in the teaching of mathematics and science, but that in many cases a definite attempt has been made to correlate this teaching.

Mathematical masters may give much valuable help to their science colleagues by setting problems requiring physical data and expressed in metric units; an excellent start in this direction has been made, and it is hoped that such problems will become more common in text-books. It is, however, pro-

* Published separately, 6d. net. (Messrs. Bell & Sons.)

bable that at the present time the chief obstacle to cooperation is that lack of laboratory training or experience which is unfortunately so common amongst mathematicians; an attempt has recently been made to remedy this, and the Committee wish to call particular attention to the vacation course in elementary practical work organised for schoolmasters at the Cavendish Laboratory, Cambridge. Details of the nature of the courses, fees, etc. will be found in Appendix C.

The Committee recognise the value of Nature Study as a training, especially for younger boys, whether it is associated with physical geography or taught in other ways; but since such teaching in Preparatory Schools has been already dealt with by the Association of Public School Science Masters,* and Mathematical Masters are not particularly concerned with it, the Committee, beyond obtaining statistics as to the conditions under which it is taught, have not considered it in the present report.

RECOMMENDATIONS.

The Committee recommend that:

1. Measurements of length with a decimal scale should be taught as soon as decimal fractions are introduced. Before proceeding to any further measurements, all boys should be familiar with the addition, subtraction, multiplication and division of decimals treated on modern lines;† they should also be familiar with unitary method; and even in cases where some other method of dealing with questions of proportion is used, care should be taken that the unitary method is not forgotten, so that examples may still be treated by it when desired.

2. Practical work should be taught as early as possible in Preparatory Schools, and from the lowest forms of Public Schools; the first introduction to it should be in practical geometry, including the measurement of lines, angles, and areas of both plane and solid figures, and the volumes of the latter.‡ The actual measurement of solid models is extremely important, and boys should be taught to make free-hand sketches of them. (Although for class purposes it will be found advisable to use wooden models, it may be pointed out that it is a valuable exercise for the boys themselves to construct models of the simpler solids.)

3. The measurements suggested in recommendation 2 should be entirely in the hands of the Masters who teach mathematics,

* See *Preparatory Schools Review*; March, 1905.

† See The Mathematical Association report upon *The Teaching of Mathematics in Preparatory Schools*. Price 3d. (Geo. Bell & Sons.)

‡ This being approached by building up rectangular blocks with inch or centimetre cubes.

and in no case delegated to either the Science or the Art Staff. The work may be done in an ordinary mathematical class room (sets of models being kept in the room) and should form an essential part of the mathematical teaching; *e.g.* when the subject of areas is approached in arithmetic, the first question should be such as the following: "Draw a rectangle 4 inches long and 3 inches wide; divide it into square inches; of how many square inches does the rectangle consist?" hence lead up to the method of finding the area of a rectangle. The boys should then, from their own measurements, find the area of the faces of rectangular models, the areas of rooms and, where possible, of playgrounds, etc. Until this has been done, the abstract questions of the text-books should be avoided.

There seems to be, at the present time, a special danger of the practical drawing being considered an end in itself. At this stage, its main aim should be to lead up to and help in other work; of course its special use is to aid by a process of induction the acquisition of the fundamental facts dealt with in deductive geometry; *e.g.* the construction of triangles from suitable data should be used to lead up to the cases of congruent triangles.

4. When the pupils have become accustomed to the measurements of rectilinear figures, they should be introduced to the method of finding the areas of irregular figures by means of squared paper, and in this connection the use of maps drawn to scale on such paper will be found a valuable means of sustaining interest. This work should be soon followed by the experimental determination of the value of π; the boys will then be in a position to use squared paper for finding the areas of circles, etc., and to understand the formulae used to express them.

5. It is desirable that, where possible, the elementary measurements of mass (or weight) and volume, including the use of scales and water, should be undertaken by the Mathematical Staff, since if it be done by the Science Staff a want of correlation with the mathematical work is likely to result. The boys, having gained an insight into the measurement of areas and volumes, should be required to generalise their ideas of the latter by measuring the sides of rectilinear solids, and then, by dropping them into a graduated cylinder containing water, to determine the displacement produced. From rectilinear solids to cylinders and spheres the transition will be a simple one for those boys who have passed through the course proposed in recommendation 4.

This work should be closely correlated with the actual working of numerical problems, *e.g.* having weighed a piece of metal and

determined its volume, the pupil should be required to calculate the weight of other solids of given dimensions and made of the same material; questions of cost, etc., may well be introduced, and an experienced teacher will have no difficulty in making his boys realize that the measurements made have a real connection, not only with the ordinary arithmetical lessons, but also with the actual details of daily life.

6. Local conditions must decide whether the measurements proposed in recommendation 5 are done in an ordinary class-room, or in a room specially devoted to practical work; but the Committee wish to point out that they can be made in an ordinary class-room without the use of bulky or expensive apparatus. It is never necessary, and generally inadvisable, for beginners to use delicate and therefore expensive balances; excellent results can be obtained by the use of ordinary apothecary's scales, if these are provided with a hook under each pan, and the pans supported, on the work-table or otherwise, each time a change is made in the weights; such scales can be bought for 3s. 6d., and are easily packed into a small space when not in use. One considerable advantage of carrying on the work in an ordinary class-room is that it is then more easily coordinated with the mathematical lessons.

The course of measurements including the use of balances should not be too long, and need very seldom exceed 20 hours of practical work; a suggested syllabus will be found in Appendix D.

7. The term "measurements" as used above should not be understood as including any form of mechanics; *e.g.* even simple experiments upon the principle of moments or the parallelogram of forces should be postponed until the pupils have gained some acquaintance with the elements of trigonometry as proposed in recommendation 9.

8. The introduction of Archimedes' principle marks a break in the teaching of practical measurements, and when this point is reached the work may be left to either the Mathematical or the Science Staff.

9. It is so important that sufficient time be devoted to arithmetical exercise, that it is in general inadvisable for boys to begin the study of trigonometry while at Preparatory Schools. If, however, they do make a start, the work should be confined to numerical trigonometry, *i.e.* the pupils should be taught the meaning of the ratios, the method of calculating them, and their use in the solution of triangles by division into right-angled triangles. But all questions of identities should be left until the boys are at Public Schools, in which numerical trigo-

nometry as defined above should be commenced as early as possible, and form the first term's work in this subject.

10. The aim of mechanics teaching in the earlier stages should be to familiarise the pupil with mechanical principles rather than to enable him to solve mathematical problems. At first, therefore, he should be made to rely upon experimental and graphical methods. As the course advances trigonometry should be introduced into the work, and in order that this may be ready to hand, a short course of trigonometry, as recommended above, should precede any instruction in mechanics.

It is advised that statics be begun in the lower part of the "Upper School" as a part of the regular mathematical teaching, *i.e.* it should be taught by the Mathematical Master during mathematical hours. The first term should be devoted to practical work in the laboratory, and comparatively simple numerical and graphical applications of the principles arrived at by experiment, the more formally mathematical portions of the work being left to be dealt with in the subsequent terms.

11. The use of logarithms is at some Schools introduced as a branch of arithmetic, and the results obtained prove that this plan works well: the more usual method, however, seems to be to introduce the subject immediately after the pupil has learned the meaning of a fractional index in algebra. If the latter method be adopted, it is important that those boys who are backward in mathematics shall not be allowed to spend an undue amount of time in working at simultaneous quadratic equations, etc., before the use of logarithms is explained to them.

The Committee recognise that there are several methods by which the meaning of a logarithm may be made clear to boys, and they do not wish to urge any one method to the exclusion of others; they believe, however, that by whatever method they are taught, the object should be to enable boys to use four-figure tables as an instrument of calculation, with as much idea of the underlying principles as they seem able to acquire without difficulty. Beginners should never be provided with tables of anti-logarithms or co-logarithms.

12. Whilst fully realizing that arithmetical accuracy is of the greatest importance, and that the younger boys must be drilled until this is attained, the Committee regret to find that there is only a small percentage of schools in which the use of the slide rule is taught, and that even in these the instruction is confined to the senior boys. They recommend that its use be explained to all boys as soon as they have mastered the use of logarithms; a few slide rules should be considered to be a necessary part of the equipment of all physical laboratories, and the boys encouraged to use them as much as possible.

13. Elementary physics should always be introduced before any chemistry is taught; the laws of heat form an essential introduction to the study of chemistry, and the Committee recommend that when possible at least two terms should be devoted to their study before beginning chemistry.

It is of the utmost importance that in teaching the early stages of elementary heat, the master should neither use nor allow the use of formulae; *e.g.* when treating problems on the expansion of solids, the boy should be made to understand that the coefficient of linear expansion is the amount by which unit length expands when heated through unit temperature, and should then make the calculation by ordinary unitary methods.

Example.

A brass rod is 25 metres long at 10° C.: find its length at 50° C. if the coefficient of linear expansion of brass is ·000018.

1 metre of brass heated			1° C. expands		·000018 metres.
25	„	„	1° C.	„	·000018 × 25 metres.
25	„	„	40° C.	„	·000018 × 25 × 40 metres
					= ·018.

Length at 50° C. = 25·018 metres.

Again, when dealing with simple problems upon calorimetry, the pupil should be made to calculate the number of calories gained by the water (and the calorimeter), to realize that the whole of this has come from the body under examination, and to use the result to find how many calories were lost by one gram of the substance falling one degree.

Example.

A calorimeter weighing 60 grams and at a temperature of 20° C. has 50 grams of water at 100° C. poured into it, and the resulting temperature is found to be 92·2° C. Calculate the specific heat of the material of which the calorimeter was made.

1 gram of water falling			1° C. gives up		1 calorie.
50	„	„	7·8° C.	„	50 × 7·8 calories
					= 390 calories.
60 grams of substance rising			72·2° C. requires		390 calories.
1	„	„	1° C.	„	$\frac{390}{60 \times 72{\cdot}2}$ calories
					= ·09 calories.

Shorter methods may be introduced as the pupil becomes more experienced, but in any case the premature use of a formula such as

$$S = \frac{m(T-t)}{M(t'-T)},$$

before the physical facts upon which it is based are clearly grasped, only serves to reduce a valuable educational lesson to mere cram.

14. It is undesirable that either formal physics or chemistry be taught in Preparatory Schools, and it is suggested that in the few schools where at present either of these subjects is taught, the time might be more profitably devoted to practical measurements. Questions should not be set in formal physics or chemistry at the entrance, or entrance scholarship examinations to the Public Schools.

APPENDIX A.

COPY OF CIRCULAR SENT TO SCHOOLS ASKING FOR INFORMATION AS TO THE PRESENT CONDITIONS UNDER WHICH MATHEMATICS AND SCIENCE ARE TAUGHT.

THE COLLEGE, MALVERN, 1*st March*, 1909.

DEAR SIR,—During the past year the Mathematical Association and the Association of Public School Science Masters have been considering how it is possible to produce a closer co-operation between the teachers of mathematics and science than exists at present. At the separate meetings of the two Associations held on 12th January last it was decided to hold a joint meeting in 1910 to consider the question, and a committee consisting of members of each, with two representatives of the Association of Head-Masters in Preparatory Schools, has been appointed to collect information and to present a report, which will be laid before the joint meeting.

I am requested to send you the enclosed circular, and to ask if you will be kind enough to give the information asked for; in publishing the report no names of schools will be mentioned, and of course all information will be treated as strictly confidential.

As the circular is being sent to all the Secondary Schools in England, it may be found that some of the questions relate to subjects not taught in your school; should this be the case, it will greatly reduce the work of tabulating the answers if you will kindly cross out such questions. It will also be a help if when indicating "position in school" you will use such expressions as "Top Form," "Middle Forms," etc., rather than mention them by name.

Yours very truly,

DOUGLAS P. BERRIDGE

(*Hon. Sec. to Committee*).

Name of School ....

Nature of School (Preparatory, Grammar, or Public) ....

Number of Boys now on the books ....

1. Kindly state at what age and position in the School boys begin the study of the following subjects, also give the percentage of boys who AT ANY PERIOD OF THEIR SCHOOL LIFE take a course in each :—

(*a*) Practical Geometry, *i.e.* accurate construction with rule and compasses.

Age ...... Position in School .... Percentage ......

(*b*) Practical Measurements of Length, Area, and Volume, without the use of balances or water.

Age ...... Position in School .... Percentage ......

(*c*) Practical Measurements of weight, etc., using a balance, but not water.
Age ...... Position in School .... Percentage ......

(*d*) Practical Measurements requiring the use of balances and water.
Age ...... Position in School .... Percentage ......

(*e*) Nature Study, including practical work.
Age ...... Position in School .... Percentage ......
(*Please state the nature of practical work in Nature Study.*)
....
....

(*f*) Practical Physics (*i.e.* Heat, Optics, Electricity).
Age ...... Position in School .... Percentage ......

(*g*) Practical Chemistry.
Age ...... Position in School .... Percentage ......

(*h*) Statics.
Age ...... Position in School .... Percentage ......

(*i*) Dynamics.
Age ...... Position in School .... Percentage ......

2. Are the subjects marked (*a*), (*b*), (*c*), (*d*), (*e*), (*h*), (*i*) taught by Mathematical or Science Masters? If one Master teaches both Mathematics and Science, please write "combined"; if the boys are taught by two separate masters, of whom one is a Science Master, the other a Mathematical Master, please write "both."

(*a*) .... (*e*) ....
(*b*) .... (*h*) ....
(*c*) .... (*i*) ....
(*d*) ....

3. Are the subjects marked (*a*), (*b*), (*c*), (*d*), (*h*), (*i*), taught in an ordinary class-room, in a physical laboratory, or in a mathematical laboratory?

(*a*) .... (*d*) ....
(*b*) .... (*h*) ....
(*c*) .... (*i*) ....

4. Are the measurements described under (*d*) confined to the use of the specific gravity bottle, or is Archimedes' principle used?

....

5. Are the subjects mentioned in question 1 taught by forms, by mathematical sets, or by science sets?

(*a*) .... (*f*) ....
(*b*) .... (*g*) ....
(*c*) .... (*h*) ....
(*d*) .... (*i*) ....
(*e*) ....

6. How many hours each week are devoted in each part of the school

(*a*) in school to mathematics? Lower ...... Middle ...... Upper ......

(*b*) during preparation to mathematics? Lower ...... Middle ...... Upper ......

(*c*) in school to science lectures? Lower ...... Middle ...... Upper ......

(*d*) in school to practical science? Lower ...... Middle ...... Upper ......

(*e*) during preparation to science? Lower ...... Middle ...... Upper ......

7. At what stage is the USE (not theory) of logarithms taught? Is it generally taught by the Science or the Mathematical Master?

....

8. Is the use of the slide rule taught? If so, at what stage?

....

9. What proportion of the time given to (*a*) Statics, (*b*) Dynamics, is devoted to practical work?

Statics .... Dynamics ....

10. Is the mathematics of candidates for science scholarships taught by the Mathematical Staff or by the Science Staff?

....

General Remarks.

Signed ....

APPENDIX B.

SUMMARY OF RETURNS SHOWING THE PRESENT CONDITIONS UNDER WHICH PRACTICAL MATHEMATICS AND SCIENCE ARE TAUGHT.

Copies of the circular reproduced in Appendix A. were sent to all schools mentioned in *The Schoolmasters' Year Book*, the total number posted being 1380; since, however, those schools represented on the Association of Public School Science Masters received two copies, one being addressed to the Senior Mathematical Master and one to the Senior Science Master, the total number of schools to which application was made was only about 1300.

Two schools replied that they did so little practical work that any return they could make would be useless, two others wrote saying that the schools were now closed, and three stated that it would entail too much trouble to furnish the information; 279 forms were filled in and returned.

A considerable difficulty was found in making the separation between "Public Schools" and "Grammar Schools"; on the one hand many well known schools which are represented on the Headmasters' Conference retain the title of "Grammar School," under which they have become famous, on the other a large number of small modern schools which draw their pupils entirely from their own neighbourhood prefer to call themselves "Public Schools." The Association of Public School Science Masters defines a Public School as one "which is represented on the Headmasters' Conference, or which, in the opinion of the Committee, is of similar status to the schools so represented"; and in the following summary this definition has been used. Those schools represented on either the Headmasters' Conference, or the Association of Public School Science Masters being classed as "Public Schools," whilst all others which do not describe themselves as being "Preparatory" are classed as "Grammar Schools."

Using the definition given above there are, upon a rough estimate, about 100 Public Schools, 800 Grammar Schools, and 400 Preparatory Schools. Of the replies received 55 came from Public, 175 from Grammar, and 49 from Preparatory Schools. It will be noticed that the percentage of the latter making returns is much smaller than in either of the other cases; this may be partly accounted for by the Headmasters considering that the circular did not affect their schools; it seems, however, probable that very little practical work is at present done in the majority of Preparatory Schools.

PRACTICAL GEOMETRY.

This subject is taught in more than 98 per cent. of the schools, and with very few exceptions every boy passes through a course during some period of his school life. It is an almost universal practice to begin teaching it in the bottom form, and consequently the age of the pupils varies with that of entry; in Public Schools the average is 13, in Grammar Schools 11, and in Preparatory Schools 9.

In the majority of schools practical geometry is taught by the Mathematical Staff, but in 10 per cent. of Grammar Schools and in nearly 2 per cent. of Public Schools it is taken by the Science Master, whilst in about 4 per cent. of both Grammar and Public Schools, the lessons are given by the Art Master; no numbers are available in the case of Preparatory Schools, since it is unusual for masters in these to confine their teaching to special subjects.

In 15 per cent. of the Grammar Schools and in 14 per cent. of the Public Schools, the work is done in laboratories; when it is taught by the Art Master the classes are taken in the Art Room, and in all other cases an ordinary class-room is used.

PRACTICAL MEASUREMENTS.

The questions in the circular were framed with the view of determining whether any considerable number of schools found a difficulty in introducing the use of balances and water; the returns prove that this is not the case, practically no distinction being made between measurements requiring and those not requiring their use. In many cases the average age at which measurements with balances are commenced is returned as six months higher, and the age at which the use of water is introduced as twelve months higher, than for measurements not requiring them. It seems probable, however, that this simply means that the full course extends over more than a year, and that the pupils begin with the simpler experiments.

It is the universal custom in both Public and Grammar Schools, for a course in practical measurements to precede any lectures or practical work in formal physics; the average age for beginning the subject is 12·9 years in Public Schools and 11·6 years in Grammar Schools. Only a small proportion of Preparatory Schools appear to teach measurements, but in those which do the average age for beginning is 11 years.

89 per cent. of Grammar Schools and 55 per cent. of Public Schools teach elementary measurements to every boy during some portion of his school life; in very few schools does the percentage of boys who fail to do any measurements rise to 50.

The teaching is chiefly in the hands of the Science Staff, but measurements not requiring the use of balances or water are taught by Mathematical Masters in 11·4 per cent. of the Public and in 15·5 per cent. of the Grammar Schools. As would be expected under these circumstances, it is unusual to find the work done out of a laboratory, and this is the case in only 4 Grammar and 1 Public School; Preparatory Schools, on the other hand are seldom provided with laboratories, and no difficulty seems to be experienced by them in teaching measurements in ordinary class-rooms.

Archimedes' principle is all but universally used in the practical dermination of the volumes of irregular solids, and of specific gravities.

NATURE STUDY.

In analysing the returns received, it was necessary to distinguish between Nature Study taught as a definite school subject, and mere membership of a Natural History Society. Almost all Public Schools, and a very large

number of Grammar Schools, have such societies; since, however, the membership is voluntary and the work is done entirely out of school hours, the instruction given in connection with them is not included in the following summary, which refers only to Nature Study forming an actual part of the school curriculum.

The proportion of schools teaching Nature Study (about 43 per cent.) is almost the same for both Public and Grammar Schools; it is always commenced in, and generally confined to, the lowest part of the school, the average age being 11·5 years in Public and 9 years in Grammar Schools; these ages prove that it is looked upon as being chiefly suitable for preparatory departments, and in very few cases is any attempt made to continue the teaching when the pupil rises to the higher parts of the school, nor does it seem to be thought necessary to make provision for those boys who on entry are placed in higher forms. In Dual Schools, however, it is a general practice for the girls to continue Nature Study, whilst the boys take a course in physics.

In a few cases special masters are engaged to teach Nature Study, or it is in the hands of a mistress (presumably one who teaches form subjects to the younger children), but in the majority of schools the lessons are taken by the science masters.

Out of the 49 Preparatory Schools making returns this subject is taught in only 8, and even in these it does not seem to be treated as a very serious branch of education.

A considerable number of schools make no provision for regular practical work in connection with Nature Study, but it is satisfactory to find that in the majority of cases experiments are made; the practical work is however of the most varied nature, ranging from "School Gardening" or "Rambles and Records," to one school where at the average age of 8 years, the children are instructed in "Elementary Morphology and Physiology, including the use of the microscope."

PRACTICAL PHYSICS AND CHEMISTRY.

These subjects are taught in all the Public Schools sending replies, whilst of the 175 Grammar Schools which make returns Practical Physics (*i.e.* Heat, etc.) is taught in all but 29, and Practical Chemistry in all but 9. In Preparatory Schools it is the exception for either subject to be taught, but five teach practical chemistry and two practical physics; of the five teaching chemistry only one teaches physics as well, whilst in three no provision seems to be made for measurements.

The results of the present investigation are in marked disagreement with those obtained by the British Association Committee in 1908; since the interval of time between the two is too short for it to be possible that any marked change has taken place in the teaching, it is probable that the difference is due to the B.A. Committee having dealt with many less schools than has the present investigation.

In 27 per cent. of the Public Schools, and in 34 per cent. of Grammar Schools, chemistry is begun before physics; in 16 per cent. of Public and 23 per cent. of Grammar Schools elementary physics precedes chemistry; in the remainder both subjects are started at about the same time. The average age for beginning chemistry is 14·6 for Public and 13·2 for Grammar Schools; the age at which physics is commenced being 14·9 and 13·6 for the respective classes of school.

Chemistry is taught to 100 per cent. of the boys in the large majority of Grammar Schools, the number in which it is taught to less than 50 per cent. being very small. In Public Schools, on the other hand, the percentage is much lower; in only twelve cases do all the boys learn it, and the average

number who learn chemistry during some portion of their school life is probably about 50 per cent. of the total in the school.*

It seems to be universal to teach Heat before either Optics or Electricity, and in a considerable number of Grammar Schools the whole instruction in physics is confined to Heat; in the Public Schools it is the exception for boys to take a course in Heat, and not proceed to either Optics or Electricity in their second year.

MECHANICS.

In no subject is the want of co-operation between the Mathematical and the Science Masters so apparent as in mechanics. In the large majority of schools mechanics is taught by the Mathematical Master as a part of the regular mathematical work; no experiments are made, and in very few cases are practical difficulties mentioned; this instruction is confined to the senior boys, with the result that the Science Master finds the boys who come to him are ignorant of such conceptions as the principle of moments, or the parallelogram of forces, and unable to distinguish between "force," "work" and "power," and is obliged to put most of his younger boys through a short and necessarily inadequate course in practical mechanics, in order that they may be able to follow his lectures. There is no connection between the lessons given by the two masters, and it too frequently happens that the better mathematicians leave school without having done any practical work in applied mathematics, whilst many promising science boys are deprived of the valuable training afforded by the mathematical treatment of this subject.

The requirements of the Army Entrance Examination have caused a certain amount of practical work to be done during the past few years, but even now rather less than half the Public Schools (which are those chiefly affected by the regulations of the Civil Service Commissioners) report that practical work forms a part of the regular teaching in mechanics.

THE USE OF LOGARITHMS AND THE SLIDE-RULE.

A few schools teach the use of logarithms as a branch of arithmetic and report that the plan works well; in the great majority of cases, however, the subject is not mentioned until the pupil has reached fractional indices in algebra. Naturally under these conditions, the teaching is almost entirely in the hands of the Mathematical staff, and the boys do not begin to use either logarithms or the slide-rule until they are in the upper part of the school. It seems probable, that the use of logs, as a means of simplifying ordinary arithmetical calculations, is only incidentally touched upon, and in the majority of cases, pupils are still taught to look upon them as being useful chiefly in trigonometrical and in advanced arithmetical work.

The slide-rule seems to be by no means in general use, and although 66 per cent. of the Public Schools explain its use to the senior boys, this is only done in 28 per cent. of the Grammar Schools. There can be but little doubt that this difference between the two classes of school is due to the initial cost of the necessary instruments, for the great majority of Grammar Schools in which its use is taught, add a note that the instruction is confined to those boys who provide themselves with rules. Since now cheap slide-rules (without trigonometrical ratios) have been placed on the market, it is probable that its use will become much more common.

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*Mr. O. H. Latter states in the "Report on Science Teaching in Public Schools Represented on the Association of Public School Science Masters" that "It would probably be not far wide of the truth to state that of those who pass through our Public Schools at the present time about 90 per cent. are compelled to take a longer or shorter course of science"; he is there, however, referring to all branches of science, and even those boys who take only a short course in practical measurements are included; the number who learn any physics or chemistry is much smaller.

TABLE SHOWING THE AVERAGE AGE AT WHICH EACH SUBJECT IS BEGUN.

	Public Schools.	Grammar Schools.	Preparatory.
Practical Geometry	13	11	9
Measurements without Balances	12·9	11·6	11
Measurements with Balances and Water	13·6	12·4	
Nature Study	11·5	9	
Practical Chemistry	14·6	13·2	
Practical Physics	14·9	13·6	

TABLE SHOWING THE PERCENTAGE OF THE VARIOUS MASTERS WHO TEACH THE DIFFERENT SUBJECTS.

	Public Schools.					Grammar Schools.				
	Practical Geometry.	Measurements; no balance.	Measurements and balance.	Statics.	Dynamics.	Practical Geometry.	Measurements; no balance.	Measurements and balance.	Statics.	Dynamics.
Mathematical	64·1	15·5	7·7	37·7	42·3	46·8	11·4	0·6	24	24
Science	1·9	37·7	61·5	7·5	3·8	10	39·3	57	24	21
"Combined"	17·0	31·0	25·0	18·8	17·3	33·1	31·4	28·8	34	29
"Both"	13·2	15·5	5·7	35·8	36·5	—	17·9	13·5	20	26
Art	3·8	—	—	—	—	4·3	—	—	—	—
"Form"	—	—	—	—	—	5·8	—	—	—	—

APPENDIX C.

DETAILS OF THE LONG VACATION COURSE IN PHYSICS FOR ASSISTANT MASTERS HELD AT CAMBRIDGE IN 1909.

A course in practical physics was held in the Cavendish Laboratory, by permission of Prof. Sir J. J. Thomson, during the first three weeks of August; it was open to all assistant masters, and the number joining was limited to twenty.

The laboratory fee for the three weeks was £3 3s., and since it is possible to obtain very comfortable rooms with board in Cambridge during the "Long" at a charge of from 30s. to 40s. per week, it will be seen that the total cost of the three weeks was not more than about £10.

Facilities were arranged for obtaining recreation in the form of golf, cricket, tennis and rowing.

The course was not designed as a preparation for any examination, but the following list of experiments which were available to those joining was published about a month before the course began; since it was impossible for any one master to perform more than about twenty experiments, all sending in their names were asked to select those they wished to do, and to send in the list a short time before the meeting.

MECHANICS AND THE PROPERTIES OF MATTER.

1. Determination of g by the pendulum.
2. Rigid pendulum.
3. Ballistic ba'ance.
4. Diagram of forces.
5. Funicular polygon.
6. Harmonic motion of mass hung on helical spring.
7. Comparison of moments of inertia.
8. Young's modulus by stretching.
9. Rigidity.
10. Surface tension by capillary tube.
11. Velocity of sound in air.
12. Resonance experiments.

HEAT.

13. Air thermometer.
14. Water thermometer.
15. Coefficient of linear expansion.
16. Specific heat of solid.
17. Latent heat of steam and water.
18. Melting point of wax by method of cooling.
19. Thermal conductivity of indiarubber.
20. Mechanical equivalent of heat by frictional experiment.

LIGHT.

21. Snell's sine law.
22. Refractive index by method of total reflexion.
23. Experiments with thin lenses.
24. Experiments with mirrors.
25. Angle and refractive index of prism by spectrometer.
26. Wave lengths by diffraction grating.
27. Experiments with thick lenses and systems of lenses.
28. Focal lines formed by reflexion at concave mirrors or by astigmatic lens.

ELECTRICITY AND MAGNETISM.

29. Comparison of magnetic fields by vibration magnetometer.
30. Pole strength by Robinson magnetometer.
31. Determination of the magnetic moment of a magnet and of the horizontal component of the earth's field.
32. Magnetization of iron.
33. Reduction factor of tangent galvanometer by copper voltameter.
34. Measurement of resistance by tangent galvanometer.
35. " " wire bridge.
36. " " post office box.
37. " " Carey Foster's method.
38. Potentiometer.
39. Determination of J by heat produced in a coil.
40. Experiments on induction of currents.
41. Comparison of capacities.
42. E.M.F. of thermo-electric couple.

Further particulars may be obtained from G. F. C. Searle, Esq., F.R.S., Cambridge, or from F. S. Scruby, Esq., Aldenham School, Elstree, Herts.

APPENDIX D.

SYLLABUS FOR A SHORT COURSE IN PRACTICAL MEASUREMENTS.

1. Measurement of straight lines by means of scale, including the estimation of tenths of the smallest scale division.

2. Measurement of the diameter of cylinder and sphere by placing between two rectangular blocks of wood and measuring the distance of these apart; also by means of calipers (without vernier).

3. Measurement of curved lines, and determination of the value of π (see Appendix E).

4.*Examination of the micrometer screw-gauge, and application of its use in the measurement of diameter of wire, etc.

5.*Examination of the vernier, and application of the instrument to slide calipers; use of the latter, especially for finding the diameter of small spheres, etc.

6.*Examination and use of the spherometer.

7. Measurement of the areas of rectangles and triangles by means of squared paper, and determination of the formulae to express these areas.

8. Measurement of areas of plane surfaces of irregular outline by means of squared paper.

9. Area of circles by means of squared paper, and confirmation of formula.

10. Determination of the formula for volumes of rectangular solids, special stress being laid upon the fact that this is the product of the area of cross section and the length.

11. Practice in reading convex and concave menisci.

12. Measurement of the volumes of dense insoluble solids by displacement of water in graduated cylinders or burettes.

13. Determination of the formulae for the volumes of non-rectangular solids by methods 10 and 12.

14.*Measurement of inaccessible heights and distances, and finding the areas by triangulation, etc., the results being determined by drawing to scale.

15. Examination and use of balance and weights.

16. Determination of areas by cutting out in cardboard and weighing.

17. Determination of the weight of unit volume of various substances.†

18. Determination of the weight of 1 c.c. of water, and elementary ideas of the meaning of specific gravity.

19. The specific gravity bottle, and its use in finding the specific gravity of liquids.

20. The specific gravity of solids by means of the specific gravity bottle.

21. Proof of Archimedes' principle by "bucket and cylinder experiment."

22. " " " method of displacement.

23. Experiments on flotation, *e.g.* floating a weighted rod upright in water, weighing the rod, measuring the volume immersed, and hence finding the weight of water displaced.

24. The principle of the common hydrometer, approached by repeating experiment 23 with alcohol instead of water.

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* The exercises marked * may be introduced in the order given or later at the discretion of the master.

† Although this value is generally called "density," it is better to avoid using the word with beginners.

25. Specific gravity of dense solids by Archimedes' principle.
26. „ „ liquids „ „
27. „ „ solids less dense than water by Archimedes' principle.
28. Specific gravity of solids soluble in water by Archimedes' principle.
29. Experiments with liquids in U-tubes, and in communicating vessels.
30. Determination of the relative density of two liquids which do not mix, using U-tubes.
31. Density of liquids which do mix, using U-tubes and mercury, or Y-tubes.
32. Density of liquids by Hare's apparatus.
33. Determination of the diameter of tubes by partly filling with mercury and weighing.
34. Boyle's law, etc.

APPENDIX E.

SUGGESTED METHODS FOR THE EXPERIMENTAL DETERMINATION OF THE VALUE OF π.

ANY of the following methods for finding the value of π are recommended; in the majority of cases it is wise for the pupils to find it by at least two separate methods.

A. The diameter of a cylinder is found either by means of calipers or by placing it between two rectangular blocks of wood and measuring the distance of these apart; a piece of thin paper is then wrapped round the cylinder, pricked with a pin, and the circumference found by measuring the distance between the two resulting holes after the paper has been unfolded. Several cylinders of different diameters should be used, in order that the pupil may become convinced of the constancy of the ratio.

B. A series of circles is drawn, the diameter of each measured directly, and the circumference approximately found by stepping round the circle by dividers.

C. Circles are drawn and their diameters measured as in B, but the circumference measured by means of an opisometer or by placing the end of a piece of thread upon one point and carefully tracing round the circle by its means.

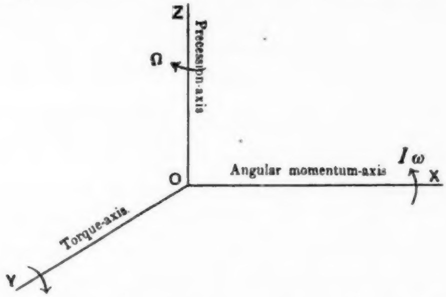
D. A penny or other large coin is placed upon a sheet of paper against a straight edge; a pencil mark is made upon the coin at its lowest point, and also upon the paper at this point; the coin is now rolled along until the pencil mark is once more in contact with the paper, when another mark is made; the distance between the marks is measured directly, and the diameter of the coin found by means of calipers.

E. A cylinder is placed on its end, a mark made at any point on its circumference and this placed against a scale; by means of another rule the cylinder is rolled along the scale until the mark is once more in contact with the latter: having obtained the circumference by this means, the diameter is directly measured.

MATHEMATICAL NOTES.

299. [R. 8. β.] *Euler's Equations deduced from Gyroscopic Resistance.*

(Rotations which are *left-handed* when viewed from the origin are considered *positive.*)



If a body is rotating with angular momentum $I\omega$ about OX, and a torque K be applied about a perpendicular axis OY, then it can be proved* by the parallelogram of angular momenta that the axis of angular momentum (or more shortly, the momentum-axis) is rotated, or "precesses," about the *third* perpendicular axis OZ with angular velocity Ω determined by the equation

$$K = I\omega\Omega.$$

It should be remembered that this equation only refers to existing motion and not to the *start* of precessional motion.

Since K produces no visible effect about its own axis, it is clear that the quantity $I\omega\Omega$ represents the resistance which the body offers to being turned about OY, frequently called the *gyroscopic resistance* of the body.

The rule of the direction of precession may be stated thus: If the momentum-axis and the torque-axis are drawn *in the same sense*, the momentum-axis *sets itself towards* the torque-axis.

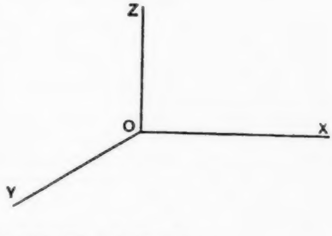
General equations of motion of a body referred to three moving axes.

If OX, OY, OZ are three moving axes mutually at right angles but not necessarily fixed in the body.

ω_1, ω_2, ω_3 the angular velocities of the body at any instant about these axes.

h_1, h_2, h_3 the components of angular momentum of the body.

θ_1, θ_2, θ_3 the rates at which the axes are turning about axes fixed in space with which they are instantaneously coincident, then we see that the angular momentum



h_1 is precessing about OZ with velocity θ_3,

and " " OY " θ_2.

Similarly for the other components h_2 and h_3.

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*A proof of this theorem (with ω and Ω interchanged) is given in Professor Worthington's *Dynamics of Rotation*, on page 146 *sqq.*

Hence if a torque K_2 is applied to the body about OY, we see that the gyroscopic resistance of the body to being turned about OY is

due to the component h_1, measured by $h_1\theta_3$,
and " " h_3, " $-h_3\theta_1$,

the negative sign being taken since θ_1 tends to turn the angular momentum h_3 *away from* the torque-axis OY.

Hence we have
$$K_2 - h_1\theta_3 + h_3\theta_1 = \dot{h}_2,$$
$$\text{or} \quad K_2 = \dot{h}_2 - h_3\theta_1 + h_1\theta_3$$
and two similar equations, if we suppose torques K_1 and K_3 to act about OX and OZ respectively.

These general equations were originally given in this form by Mr. R. B. Hayward, F.R.S., of St. John's College, Cambridge, his method of obtaining them being first published in 1856 in Part I. Vol. X. of the *Cambridge Philosophical Transactions.*

Euler's equations can of course be deduced from the above. For when the axes are fixed in the body we have $\theta_1 = \omega_1$, $\theta_2 = \omega_2$, $\theta_3 = \omega_3$; and if further the three moving axes are the three principal axes of the body, the respective moments of inertia being A, B, C, the above equations reduce to
$$K_1 = A\dot{\omega}_1 - (B-C)\omega_2\omega_3,$$
$$K_2 = B\dot{\omega}_2 - (C-A)\omega_3\omega_1,$$
$$K_3 = C\dot{\omega}_3 - (A-B)\omega_1\omega_2.$$

H. Crabtree.

300. [K. 20. c.] *A Simple Device.*

When about to solve a trigonometrical equation, the student is often puzzled as to his best course, whether, in order to avoid introducing radicals, to express all the ratios as sines, cosines, or tangents. The following simple device, due to M. Brioche, is not, as far as we know, to be found in English text-books.

If the equation is unchanged on substituting $-\theta$ for θ, express all ratios in cosines. If unchanged on writing $\pi-\theta$ for θ, write all in sines. If unchanged on writing $\pi+\theta$ for θ, express all in terms of $\tan\theta$. If the equation is changed by each substitution, express all ratios in terms of $\tan\frac{\theta}{2}$. If each leaves the equation unchanged, express all in terms of $\cos 2\theta$. The student will see that the substitution that leaves unchanged the equation, also leaves unchanged the ratio taken as the unknown.

301. [K. 12. a.] *Constructions as tests in Examinations.*

From the centres A and B draw the tangents AP, BQ.

With AP, BQ as radii draw arcs intersecting at C and D.

CD is the radical axis of the two circles.

The above very neat construction was given without further comment in a recent examination.

Never having seen the construction before, and feeling sure that my friend could not have originated it himself, I asked him where he had got it from, expecting to be told the name of some G.D. book.

"Oh!" was the reply, "I remembered that the radical axis had something to do with the tangents, so I thought perhaps that that might be the way it was done."

This shows how valueless a construction may be as a test of geometrical knowledge when unaccompanied by any reasoning.

Cecil Hawkins.

302. [L¹. 14. a.] *On the foci of a conic inscribed in a quadrilateral.*

Mr. C. E. M'Vicker has communicated to me a very interesting and suggestive extension of the theorem due to Mr. E. P. Rouse (*Gazette*, Vol. IV., No. 73, p. 311) which may be thus stated:

"If S is the focus of the parabola inscribed in a given quadrilateral $ABCD$, and X, Y are the foci of *any* conic inscribed in the same quadrilateral, then $SX \cdot SY =$ constant; and SX, SY are equally inclined to a fixed straight line."

Let $ABCD$ be a quadrilateral whose opposite sides BA, CD intersect in F, while AD, BC intersect in G.

Then if S be the focus of the parabola inscribed in the quadrilateral, it is well known that the triangles SAB, SDC are directly similar and that the circumcircles of FBC, FAD, GAB, GCD all cointersect in S.

Moreover, the centres of all conics inscribed in the quadrilateral lie on a certain straight line,—"diameter of the quadrilateral" (Dr. C. Taylor)—which bisects AC, BD, FG.

To the single point at infinity on this diameter corresponds the centre of the inscribed parabola, and the join of the finite focus of the parabola to this point is the axis of the parabola. The axis, therefore, of the inscribed parabola is parallel to the diameter.

Let X, Y be the foci of *any* conic inscribed to the quadrilateral $ABCD$. Then the middle point of X, Y lies on the diameter of $ABCD$,—hence the quadrilaterals $ABCD$, $AXCY$, $BXDY$, $FXGY$ have all the same diameter, and consequently the axes of their inscribed parabolas have all a common direction.

Through A, C draw parallels to this direction. Then, remembering that the angle between one tangent to a parabola from an external point and a parallel to the axis is equal to the angle between the other tangent and the join to the focus, it follows that the same straight lines AS, CS determine alike by their intersection both the focus of the parabola inscribed in the quadrilateral $ABCD$ and that of the parabola inscribed in the quadrilateral $AXCY$ (for angle $XAB = YAD$).

The parabolas inscribed in the four quadrilaterals $ABCD$, $AXCY$, $BXDY$, $FXGY$ are therefore confocal. Hence the triangle SAX is similar to SYC; $SX \cdot SY = SA \cdot SC$; bisector of XSY is fixed, etc. R. F. Davis.

303. [V; R. 4.] I enclose solutions of two questions from the B. of E. Mechanics (Solids) paper, St. II., 1906. To obtain an answer, on academic lines, to question I. seemed a task of no small magnitude, for it led to a quartic with very large coefficients. Much time was spent in trying to find a simpler way until the "trick" required was found. The question, once this discovered, is (as everything else) quite simple, but as it kept in check two able mathematicians, I trust it may be of some interest to members of the Mathematical Association. Question II. also requires a special "trick," which I believe is not known as well as it deserves, as I have not come across it in any text-book. The question remains: should such questions, based upon the use of special artifices, be set in examinations, specially in examinations of that standard? They are no test of the knowledge of candidates, and merely lead them into traps from which they emerge disheartened, with a very substantial fraction of the allotted time wasted, since they cannot get full credit for fruitless efforts.*

I. Two uniform ladders are between two smooth parallel vertical walls, in a vertical plane perpendicular to the walls, with their lower extremities in contact upon a smooth horizontal plane, and their upper extremities against the walls. Show that, if their lengths are 18 feet and 32 feet respectively and the distance between the walls 33·6 feet, the two ladders will be at right angles to one another when they are in equilibrium, the weight per foot-run of the two ladders being the same.

Solution. The conditions of equilibrium give readily

$$\begin{cases} 9 \tan \theta = 16 \tan \phi, & \text{.........................(1)} \\ 16 \cos \theta + 9 \cos \phi = 16{\cdot}8, & \text{.........................(2)} \end{cases}$$

* Mr. Gheury's note suggests the propriety of establishing a "Pillory" in the *Gazette*. Contributions are invited. [Ed.]

where θ and ϕ are the inclinations of the long ladder and of the short ladder to the horizontal respectively.

Solving for $\cos\theta$, we get a quartic:

$$y=700\cos^4\theta-1470\cos^3\theta+771\cdot75\cos^2\theta-680\cdot4\cos\theta+357\cdot21=0,$$

which gives only one solution, namely $\cos\theta=0\cdot6000$ and $\theta=53\cdot7'\ 48\cdot37''$, since $\cos\theta$ cannot vary beyond $+1$ or -1. There is no question of solving the question in that way, but the solution can be very simply obtained by means of an artifice, as follows:

Writing (1) under the form $\dfrac{\tan\theta}{\tan\phi}=\dfrac{16}{9}$, one can split $\dfrac{16}{9}$ in two factors, allowing an indeterminate coefficient λ to correct the factors, if the splitting has been wrongly made; the simplest way is to split into equal factors and we have $\dfrac{\tan\theta}{\tan\phi}=\dfrac{4\sqrt{\lambda}}{3}\times\dfrac{4}{3\sqrt{\lambda}}$. This gives $\tan\theta=\dfrac{4\sqrt{\lambda}}{3}$ and $\tan\phi=\dfrac{3\sqrt{\lambda}}{4}$, consequently $\cos\theta=\dfrac{3}{\sqrt{9+16\lambda}}$ and $\cos\phi=\dfrac{4}{\sqrt{16+9\lambda}}$. Replacing in (2), and noting that $16\cdot8=\dfrac{84}{5}$, we have, after simplification by 12:

$$\frac{4}{\sqrt{9+16\lambda}}+\frac{3}{\sqrt{16+9\lambda}}=\frac{7}{5}. \quad\text{.............................(3)}$$

This equation is obviously satisfied for $\lambda=1$ or $\sqrt{\lambda}=1$, and this gives $\cos\theta=\frac{3}{5}$ and $\theta=\cos^{-1}0\cdot6$.

Moreover, there is no other solution satisfying the problem, for θ and ϕ are necessarily acute angles, $\cos\theta$ and $\cos\phi$ are necessarily positive. For real values of $\cos\theta$ and $\cos\phi$ λ must be positive, or $\lambda>0$.

For any value of $\lambda<1$ the left side of (3) will be obviously greater than the right side, while for any value of $\lambda>1$, it will be lesser than the right side. There is therefore no other value except $\lambda=1$.

II. Draw a horizontal straight line ABC, AB being 1 inch and BC 3 inches. Let ABC denote a uniform beam of weight w resting on a rough prop at B and underneath a rough prop at A. Find the direction and magnitude of the least force applied at the end C which will just begin to draw out the beam from between the props.

Solution. Let μ be the coefficient of friction of the props, P the force, θ its inclination to the horizontal, R and R' the normal reactions at B and A respectively.

Consider the moment when motion is just going to take place, the horizontal forces are in equilibrium

$$P\cos\theta-\mu R-\mu R'=0. \quad\text{.................................(1)}$$

Taking moments about A one gets

$$4P\sin\theta+R'-2w=0. \quad\text{.................................(2)}$$

Taking moments about B one gets

$$3P\sin\theta+R-w=0. \quad\text{.................................(3)}$$

These give $P=\dfrac{3\mu w}{\cos\theta+7\mu\sin\theta}$ minimum for $(\cos\theta+7\mu\sin\theta)$ a maximum.

The special artifice necessary is to multiply and divide by the square root of the sum of the squares of the coefficients of $\cos\theta$ and $\sin\theta$. The denominator becomes then $\sqrt{1+49\mu^2}\left(\dfrac{\cos\theta}{\sqrt{1+49\mu^2}}+\dfrac{7\mu\sin\theta}{\sqrt{1+49\mu^2}}\right)$.

Then if ϕ is an auxiliary angle, $\dfrac{1}{\sqrt{1+49\mu^2}}=\sin\phi$ and $\dfrac{7\mu}{\sqrt{1+49\mu^2}}=\cos\phi$, and the expression becomes $\sqrt{1+49\mu^2}[\sin(\phi+\theta)]$, maximum for $\phi+\theta=90°$.

The maximum value is $\sqrt{1+49\mu^2}$.

The minimum value of P is $\dfrac{3\mu w}{\sqrt{1+49\mu^2}}$.

Since $\phi+\theta=\frac{\pi}{2}$, $\tan\theta=\cot\phi=7\mu$ and $\theta=\tan^{-1}7\mu$.

M. E. Y. GHEURY.

304. [K. 8. 2.] *To construct a quadrilateral which shall be inscribable in a circle, having given the four sides.*

Solution.

Let x, y, z, u be the four sides.

Construct a fourth proportional to x, u, y: let it be v.

Produce a line DC (equal to z) to CE making $CE=v$.

Divide DE internally and externally in ratio $x:y$ at points P and Q.

On PQ construct a semi-circle, and in it place a chord CB equal to y.

D, C, B are three points of the required quadrilateral.

[Problem submitted by L. S. MILWARD (Malvern). Solution suggested by H. T. GILMORE.]

305. [I.] In my paper on Irrational Numbers in this *Gazette*, for January 1908, I defined an irrational number a to *be* the class of rationals of a (Cantor's) sequence which (in Cantor's theory) 'defines' it; thus $\sqrt{2}$ is the class of rationals 1, 1·4, 1·41, ... But *other* sequences $\left(\text{such as } 1,\ 1+\frac{1}{2},\ 1+\frac{1}{2}+\frac{1}{2},\ \dots\right)$ equally 'define' 2; so that I should have said either: that in the theory of irrationals we *re-define* equality so that '$a=b$' is to mean that the terms of these two sequences cannot ultimately differ by any fixed rational number, however small; or (preferably, since we need not re-define equality in it) the number $\sqrt{2}$ (for example) is the class of those rational x's such that $x^2<2$, so that every irrational *is* a Dedekind's 'Schnitt,' as Russell has maintained. I will develop these, and other, alternatives in an article for the *Messenger of Mathematics*, but it seemed proper to add this note here, as Dr. F. S. Macaulay has called my attention to the difficulty to which my short explanation (p. 205) might give rise. PHILIP E. B. JOURDAIN.

306. [X. 7.] *"The slide rule for babes and sucklings."*

To make up the multiplication table take a ruler A with a scale graduated in, say, centimetres or any other convenient units, say from 0 to 100. To write down or repeat multiplications by 3, take a second ruler B, and on it mark off divisions each equal to 3 divisions of the first ruler, and number these 0, 1, 2, 3, ... Then placing the ruler B against the ruler A, we read off 3 times 1 are 3; 3 times 2 are 6, and so on.

For multiplication by 7, we must take a ruler B in which each division is equal to 7 divisions of ruler A, and so on.

According to this plan we must use a different ruler B for each different multiplying factor.

It does not make any difference whether the divisions on the ruler A are all of the same length, provided that each division of ruler B is exactly equal to the three divisions on A standing opposite it if we are multiplying by 3 and to the seven divisions on A opposite it if we are multiplying by 7.

In the slide rule, the divisions decrease in length in such a way that when the scale B is moved a certain distance to the right each of its divisions is opposite *two* divisions of the scale A; when it has been moved a little further to the right each of its divisions is opposite *three* divisions of the scale A, and so on. Thus we are able to read off multiplications by 2, 3, or any number by simply shifting the scale B to the right instead of by substituting different scales. This practically is the principle of the slide rule.

It would be an easy exercise to construct a slide rule with large divisions to the scale extending to sufficient length to illustrate this property. To do so it would only be necessary to take the lengths of the graduations from a book of logarithms. I should think if any teacher were to show his pupils such a rule and then give them the logarithm book and tell them the secret of making one for themselves, he would arouse his pupils' curiosity and instruct them at the same time. They would then want to learn something about the book of logarithms. A little attempt to unravel a mystery often adds interest to a subject. If I had been allowed to learn Trigonometry in my own way, I would have opened the book at the calculation of π (in fact I often did so) and would have worked backwards when each theorem was shown to depend on something previous. Working forwards with no visible object in view often becomes very dull, and did so in this case.

G. H. BRYAN.

306. [R. 9. a.] *Snow sliding down a roof.*

The problem dealt with by the unnamed author is not the same as that considered by Mr. M. Vicker.

In the latter case a mass of snow is supposed to be sliding down a smooth roof, and portions to break off as they reach the edge. It is assumed that the portion falling away simply leaves the remainder without exerting any pull on it. On this assumption the acceleration at any instant of the portion remaining on the roof is rightly determined to be $g \sin a$.

In the text-book problem it is not very easy to imagine the physical conditions.

A mass of snow is supposed to be in equilibrium on a sloping roof. The upper portion commences to slip, and as it descends it continually starts fresh portions into motion after the manner of an avalanche.

It is tacitly assumed that as long as the snow is in equilibrium it is supported by the requisite friction, but that directly it is started into motion the friction is negligible.

On this assumption the text-book solution quoted is correct. C. S. J.

It is unfortunately true that in order to construct easily solved problems in dynamics it is customary to make assumptions that are quite contrary to experience. But unless these assumptions are *explicitly* stated the solutions cannot be regarded as correct, and to teach them can only do mischief. The snow problem is an instance in point. It would have been easy to make an instructive problem similar in character based upon the assumption that the coefficient of statical friction between two bodies at rest is in general greater than the coefficient of sliding friction between the same two bodies in motion. When a candidate fails to answer a question in an examination it is no justification for him that he goes and calls on the examiner the next day, and explains what he *ought* to have said in writing out his answer. It is still less justification for the propounder of a question that he leaves the explaining to somebody else. G. H. B.

ANSWERS TO QUERIES.

[37, p. 212.] The annual subscription (including postage in case of foreign journals) is appended where known.

AUSTRIA—

Archiv Mathematiky a Fysiky, ... ($\frac{1}{4}$ jährl.), Praha.

Časopis pro Pěstování Math. a Fys., ... (5 H. jährl.), Praha.

Sbornik Jednoty Českých Math. v Praze, Praha.

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Bull. périod. . . . . soc. belge de géomètres (bimensuel), Anvers.
Mathésis. Recueil math. . . . . (mensuel), Gand. (8/-)

Denmark—

Nyt Tidsskrift for Matematik, Kjöbenhavn.

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Annales sci. de l'éc. norm. sup. (mensuel), Paris. (40 frcs.)
Bulletin de math. spéciales (10 fois par an.), Paris. (6 frcs.) (? defunct.)
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Jahresbericht d. deutschen Math.-Vereinigung (2–4 H. jährl.), Leipzig.
Mathematische Annalen (¼ jährl.), Leipzig.
Verhandl. d. internat. Math.-Congresses, Leipzig.
Zeitschrift für Mathematik u. Physik (2 monatl.), Leipzig.
Zeitschrift f. math. u. naturw. Unterricht (8 H. jährl.), Leipzig.
Abhandl. zur Geschichte d. math. Wiss. (zwanglos), Leipzig.
Math.-naturwiss. Mitteilungen, Stuttgart.

Holland—

Nieuw Archief voor Wiskunde, Amsterdam.
Wiskundige Opgaven, met de oplossingen . . . ., Amsterdam.
Revue semestrielle des publ. math. (2 fasc. par an.), Paris. (7/-)
Wiskundig Tijdschrift, Culembourg. (4 fr. 50 c.)

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Mathematikai és Physikai Lapok, Budapest.
Math. és Természettudományi Értésitö, Budapest.
Math. és Természettudományi Közlemények, Budapest.

Italy—

Bollettino di bibliog. e storia delle sci. mat., Genova-Torino.
Annali di matematica, pura ed applic., Milano.
Giornale di matematiche, ..., Napoli.
Le Matematiche pure ed applicate, Città di Castello.
Rendiconti del circolo matematico, Palermo. (20 lira.)
Periodico di matematiche, per . . . ., Livorno. (9 lira.)
It Pitagora, Palermo. (18 lira.)
Revue de mathématiques, Torino.
Supplemento al Periodico . . . ., Livorno. (2.50 lira.)

Japan—

Tōkyō Sūgaku Butsurigaku Kwai Kiji, Tōkyō.

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* This could not be obtained (in Feb. 1908) by me through Messrs. Hachette.

NORWAY—
Archiv for Math. og Naturvidenskab, Kristiania.

POLAND—
Prace matematyczno-fizyczne (annual), Warszawa.
Wiadomosci matematyczne (once in 2 mos.), Warszawa. (3 kop. 60)

PORTUGAL—
Jornal de sciencias math. e. astron., Coimbra. (? defunct.)
Jornal de sciencias math., phys. e. nat., Lisboa.
Annaes Sci. da Acad. Polytech. do Porto. Coimbra. (6 francos.)

ROUMANIA—
Gazeta Matematica. Bucharest. (7 lei.)

SPAIN—
Rivista Trimestrial de Matematicas. Zaragoza. (35 pesetas.)
Annales de la Facultad da Ciencias di Zaragoza. (10 francos.)

RUSSIA—
Soobščenya .... (=Rapports .... soc. math. de Kharkov).
Math. Sbornik (=Recueil math.), Moscow.
Zapiski .... (=Mém. .... soc. nouv.-Russie ....), Odessa.

SWEDEN—
Acta mathematica, Stockholm.
Nova Acta Regiae Soc. Scientiarum, Upsala.

UNITED KINGDOM—
Proceedings of the Edinburgh Math. Soc., Edinburgh. (7/6)
Math. .... reprinted from Educ. Times, London. (5/-)
Proceedings of the London Math. Soc., London. (25/-)
Mathematical Gazette, London. (1/6 per No.)
Messenger of Mathematics, Cambridge. (12/-)
Quarterly Journal of Pure and Applied Math., London.

UNITED STATES—
American Journal of Mathematics, ..., Baltimore. ($5.50)
American Mathematical Monthly, Springfield, Mo. ($2)
Annals of Mathematics, pure and applied, Cambridge, Mass. (8/-)
Bulletin of the American Math. Society, New York, N.Y. ($5)
Transactions of the American Math. Soc., New York, N.Y.

[We have made some and will make further additions to the list compiled by Dr. Muir (*Proc. Roy. Soc. Edin.*, 1905-6). Ed.]

[**8, p. 95, Vol. IV.**] May I point out that the two stanzas given on p. 120, Vol. V., are incorrect.

In the first, the third line gives 8989 instead of 3979: the word "rivalled" should be spelt "rivall'd."

In the second, the last line gives 43384279 instead of 43383279: the word "thou" must be altered to a word of three letters. M. E. J. GHEURY.

BOOKS, ETC., RECEIVED.

Preliminary Mechanical Drawing for Schools and Evening Classes. By J. TRELEAVEN. Pp. 40. 1s. 6d. 1909. (Longmans, Green.)

The Elements of Geometry in Theory and Practice. Parts I-III. By A. E. PIERPONT. (Subject matter of Euclid i, ii, iii and iv, 1-9, 15.) Pp. xvi, 387. 3s. 1909. (Longmans, Green.)

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